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Performance analysis of queueing systems with resequencing

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Abstract

The service sector lies at the heart of industrialized nations and continues to serve as a major contributor to the world economy. Over the years, the service industry has given rise to an enormous amount of technological, scientific, and managerial challenges. Among all challenges, operational service quality, service efficiency, and the tradeoffs between the two have always been at the center of service managers' attention and are likely to be so more in the future. Queueing theory attempts to address these challenges from a mathematical perspective. Every service station of a queueing network is characterized by two major components: the external arrival process and the service process. The external arrival process governs the timing of service request arrivals to that station from outside, and the service process concerns the duration of service transactions in that station. These are then fused with a routing process among stations to form the structure of the queueing network. Since the arrival, service, and routing processes are usually stochastic by nature, the study of service networks involves probabilistic analysis, which is the subject of queueing theory. Many distributed applications, such as voice data transmission, remote computations, and database manipulations, information integrity require that data exchanges between different nodes of a system be performed in a specific order. Recently, multipath routing has received some attention in the context of both wired and wireless communication networks. By sending data packets along different paths, multipath routing can potentially help balance the traffic load and reduce congestion levels in the network, thereby resulting in lower sojourn time. Under multipath routing, since consecutive packets travel possibly along different paths from source to destination, they can easily be received out-of-sequence at the destination. If the application requires packets to be processed in the order in which they were sent, then disordered packets have to wait an additional amount of time, known as the resequencing delay, before being consumed. Packet mis-ordering occurs in the following two transmission scenarios. In the first scenario, multi-

ple (or parallel) routes between the transmitter-receiver pair are utilized to send data packets to increase the data transmission rate. However, a packet transmitted along one route may experience a time delay that is different from that along another route. Consequently, a packet that was sent by the transmitter earlier than another may arrive at the receiver later, resulting in packet mis-ordering at the receiver end. In the second scenario, packets may be lost or erroneously received due to channel degradation, congestion or any network hardware malfunction along a route, in which case they have to be retransmitted for error-free data transmissions via a retransmission scheme, such as the selective repeat automatic repeat request protocol (SR-ARQ). Retransmission of corrupted or lost packets can cause packets to be received out-of-order at the receiver as well. Note that the second scenario happens when there is one single channel between the transmitter and the receiver. In practice, many applications require that the packets are received in the same order from which they were sent. For such applications, the receiver has to buffer the mis-ordered packets in a resequencing buffer, resequence them repeatedly, and deliver them in the corrected order. This process is referred to as packet resequencing. The resequencing issue in simultaneous processing systems, where the order of customers (jobs, units, etc.) upon arrival has to be preserved upon departure, is a crucial theme in the queueing theory. The main objective of this research is to find the stationary characteristics of $M/M/3/\infty$, $M/M/\infty/\infty$ and $MAP/PH/2/\infty$ queueing systems with reordering buffer of infinite capacity.

In the $M/M/3/\infty$ customers in reordering buffer may form two separate queues and focus is given to the study of their size distribution. These two queues are labeled as queue 1 and queue 2. In queue 1 there are customers that are waiting for two customers that are still in service, while in queue 2 there are customers that are waiting for one customer that is still in service. Expressions for joint stationary distribution are obtained both in explicit form and in terms of generating functions. When the parameter of service μ is equal to one and the parameter of arrival λ is between

0.1 and 2.5, numerical examples are given for the mean number of customers in reordering buffer (RB) (queue 1 and queue 2), for the variance of number of customers in RB (queue 1 and queue 2), the coefficient of correlation between queue 1 and queue 2, between queue 1 and RB, between queue 2 and RB.
In the $M/M/\infty/\infty$ we propose a new problem statement for systems with resequencing that are modeled by multiserver queues followed with infinite resequencing buffer. Focus is given to the study of joint stationary distribution of the total number of customers in queue and total number of customers in reordering buffer. Using developed analytical methods there was obtained the system of equilibrium equations which allows recursive computation of joint stationary distribution of the total number of customers in buffer and servers and total number of customers in RB.
In $MAP/PH/2/\infty$ we have a queueing system with 2 servers, in which the capacity of the collecting buffer and the reordering buffer is infinite. The type distribution of both two servers is "the phase distribution" (PH), while the arrivals follow Markovian arrival process. We introduce a recurrent algorithm to calculate the simultaneous stationary distribution of the number of the requests at servers, in the collecting buffer and in the reordering buffer. The stationary distribution of the arrival time in the system and in the reordering buffer are calculated with the Laplace-Stieltjes transform.