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Cardiovascular System: Modelling and Optimization

Abstract

CANDIDATO: **D'Arienzo Maria Pia**

COORDINATORE: **Chiar.ma Prof.ssa Patrizia Longobardi**

TUTOR: **Chiar.mo Prof. Ciro D'Apice**

COTUTOR: **Prof.ssa Rosanna Manzo**

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Abstract

A conservation law is a partial differential equation, in which the variable is a quantity which remains constant in time, that is it cannot be created and destroyed. Thanks to the conservation laws it is possible to define models able to describe real systems in which something is stored. Fluid dynamic models, which are based on them, have a wide range of applications, because they can be used to describe blood flows, traffic evolution on street networks of big cities or on motorways of big states, data flows on telecommunication networks, flows of goods on supply chains, electric networks, etc.

The model of the blood flow in the cardiovascular system will be treated in this thesis. Because of the increasing request from the medical community of scientifically and rigorous investigations about cardiovascular diseases, which are the cause of about the 40% of death, the research in this field is very active. Most of these disturbs are connected to arteriosclerosis, which leads to ictus, hearth attacks, etc. It is a common practice, in the vascular surgery, to face vascular and coronaries diseases by doing a bypass, but sometimes it fails. So, it is very important to have a better understanding of the local hemodynamic, such that the doctor can understand how different surgeries' solutions influences blood circulation and he/she can be led to the choice of the best procedure for the specific patient. All this is possible thanks to the study realized with numerical simulations, less invasive of *in vivo* investigations, and more accurate respect to *in vitro* ones.

The development of mathematical models of the cardiovascular system hails from the latest three decades, since the first centuries

before Christ, when some of the main characters of the human history studied and analyzed blood circulation, such as Aristotele, Prassagora, Galeno, Harvey, Bernoulli, Eulero, Poiseuille, Young, Frank, Womersley.

The models currently known can be classified according to the dimension, which goes from 0-D to 3-D, the prefixed goals and the required accuracy. In 0-D models with lumped parameters, the parameters are distributed spatially in a discrete way, that is all the elastic, inertial or resistive effect is concentrated in one point and it represents the global behavior of a certain district (organ, vessel, part of a vessel). They are developed in order to simulate the dynamics of the blood flow in the entire cardiovascular system, and they often use the hydraulic-electric analogy. These models can be divided in two categories: *mono-compartment models* in which increasing levels of sophistication are used to capture the systemic response, and *multi-compartment models*, in which the different parts of the vascular system are represented as distinct components. Higher-dimension models, instead, permit the variation of the parameters in the space with continuity (so models with distributed parameters are used), and they can include the term of the convective acceleration (non-linear). One of the advantages for such models is that they can reveal the detailed pressure and the distribution of the velocity in a specific segment of the vascular network, but limited computational resources restrict the dimension of the studied domain. That is why the 1-D model is usually chosen for the study of the variations of pressure and flow along the whole length of the considered vessel. During the latest years multi-scale modeling techniques have been developed, in which 0-D models are coupled with 1-D, 2-D and/or 3-D models, to give complete representations of the cardiovascular system.

The thesis is organized as follows. The conservation laws will be introduced, which are characterized by the fact that, for smooth initial data, the solution of the Cauchy problem can have discontinuities in finite time. To have global solutions it is necessary to work in a class of discontinuous functions, and look for weak entropic solutions. This will be done in chapter 1. Since the definition of such solutions is not restrictive enough to guarantee the uniqueness of the solution for the corresponding Riemann and Cauchy problems, some admissibility conditions will be proposed, such as vanishing viscosity, entropy inequality, Lax condition.

The chapter 2 will be dedicated to a brief description of the cardiovascular system and a classification of the models of blood flow, considering different dimensions.

The attention will be focused on a specific 1-D model in chapter 3. In particular, in such model, the arteries are seen like thin, homogeneous and elastic tubes, while the blood like a homogeneous, incompressible and Newtonian fluid. The governing equations will be analyzed through the characteristic method and they will be solved using the discontinuous Galerkin method and a two-step time integration scheme of Adam-Bashfort. A linearized model will be studied too, to obtain analytic solutions. To make this model realistic, also inflow and outflow boundary conditions are considered.

In chapter 4 some numeric results are presented, using different formulations of the model and different types of networks. In particular, four optimization scenarios on artificial networks are investigated, such as the effect that truncation in a fractal network has to the flow in the root edge, the effect that adding or subtracting an edge has to the network dynamics, the effect that growth of a given network has on the dynamics when a desired total outflow is obtained and optimization of the heart rate in the event of a blockage/unblockage of an edge or of an entire subtree. In addition, the simulation of the tilt table test is performed, considering the 55-edge tree with the main arteries of the cardiovascular system, modeling the cardiac valve. Finally, an optimization scenario including the convective term in the mathematical formulation is presented, in order to optimize heart rate and terminal reflection coefficient, to obtain a desired pressure.